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Review of the Development of the Fin Optimization

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ABSTRACT

This investigation spans 200 years of fin design development, from Ingenhousz's early studies of thermal conduction to today's logically optimized, application-tailored geometries. It synthesizes results from over 50 key studies on conduction–convection interactions, geometric sophistication, material innovation, and multi-mode heat transfer. Optimal designs deliver 20-50% higher heat transfer than reported in the literature and reduce material use by 15-30% compared with traditional multi-layered arrangements. Developments such as slotted, gapped, elliptical, and airfoil fin designs continually improve thermal−hydraulic performance, as evidenced by decreases in pressure drop (in some cases, very significant) with increasing Nusselt number. The inclusion of radiation effects, wet-surface operation, and variable thermal properties has increased prediction accuracy, enabling customized solutions for high-temperature, condensation, and natural convection applications. Contemporary methods use CFD, inverse heat transfer techniques, and metaheuristic algorithms such as GA and PSO to search through large design spaces. These methods also enable the manufacturing of complex topologies and power-optimized fins. Hybrid architectures now offer unparalleled flexibility for electronic cooling, automotive waste-heat recovery, and aerospace applications. This review underscores an accelerating trend toward intelligent, adaptive fins that integrate advanced materials, embedded sensing, and AI-driven optimization, promising to transform the very notion of thermal management in compact, efficient, and sustainable systems.

Introduction

After over 200 years of research, fin theory has become a well-established field of study, marked by seminal experimental observations and theoretical developments. The first work in this area is due to Ingenhousz (1785) [1], who measured the thermal conductance of copper, brass, iron, and aluminum cylinders. All rods had the same length, diameter, and surface roughness, and were coated with wax. In the experiment, one end was placed in boiling fluid and the other in air; the melting of the wax showed that heat was transmitted. In this regard, it is demonstrated that both material properties and geometry influence heat transfer. Perhaps most promising was a significant step forward by Harper and Brown, who in 1922 published a NACA report that provided the first complete mathematical treatment of conduction and convection in a cooling fin, setting the course for fin design for decades to come. Jakob [2] analyzed studies on the exacerbation of extended surfaces as far back as 1789 [3]. This observation raises the possibility that the conceptual antecedents of fin theory may extend, perhaps indirectly, beyond the results obtained by Ingenhousz in his early experimental efforts to increase heat transfer via geometric surface modification [4]. In 1945, Gardner established general equations for the temperature-excess distribution and fin efficiency for any shape of extended surface, provided it satisfies the Murray–Gardner assumptions. He analyzed fins whose thickness is described by a power-law function of the distance along an axis perpendicular to the base surface (herein, the fin height). To solve the problem in this case, Gardner introduced what he called "the profile function" to describe the variation of fin geometry [5]:

for the straight fins

$$y = y_b \left(\frac{x}{x_b}\right)^{(1-2n)/(2-n)} \qquad (1)$$

for spines

$$y = y_b \left(\frac{x}{x_b}\right)^{-2n/(2-n)} \qquad (2)$$

Murray and Gardner proposed a set of assumptions for addressing fin problems, which can be outlined as follows [5]:
- Steady-state conditions
- One-dimensional heat condition
- Constant thermal conductivity
- Uniform cross-sectional area
- Negligible heat generation
- Constant convection heat transfer coefficient
- Negligible thermal contact resistance
- Bi number <0.1
- Negligible radiation heat transfer
- Heat transfer from the tip is negligible compared to that from the lateral surface.

The temperature at the fin base is uniform.

Literature Review

Hausen (1940) [6] outlined a step-by-step method of calculating the temperature gradient and fin efficiency for an arbitrary profile thickness fin. Focke (1942) [7] found the temperature gradient in conical and cylindrical spines, and, like Schmidt, showed the variation in spine thicknesses needed to minimize material use. However, he realized that the resulting best shape was impractical, and instead he calculated the best dimensions for cylindrical and conical spines. For triangular-profile radial fins, Smith and Sucec (1969) [8] proposed a solution to temperature excess based on the formulations of Bert (1963) [9]. Extended surfaces can be classified into several categories according to their geometry and operating conditions, as shown in Fig. 1. [10]
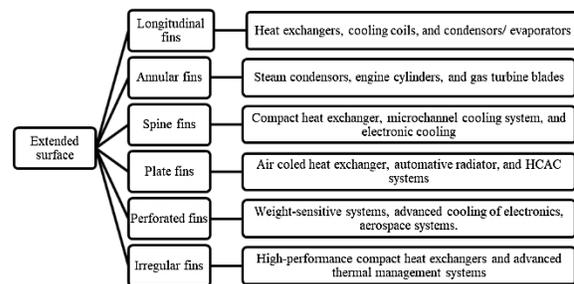

Figure (1): Extended surface classification and its applications

The design factors influencing fin effectiveness, including fin size, shape, and material, have been investigated by many researchers over the past decades. They strive to maximize heat transfer efficiency and minimize thermal resistance and manufacturing constraints. Cobble (1971) [11] provides an analytical review of fin optimisation for minimal volume and effective heat transfer. Based on

the steady-state nonlinear ordinary differential equation of heat conduction and convection, we develop an optimal fin profile that maximizes trade-offs between thermal effectiveness and material economy. The conclusions help predict optimal heat sink and fin designs across many engineering applications. A closed-form analytical result for the selection of optimal dimensions (for a fixed fin volume) of uniform annular fins that maximize heat transfer was obtained by Brown [12] in 1965. In this work, these relationships for the fin inner radius to outer radius, and thickness (δ) are determined with heat transfer coefficient and thermal conductivity to be as follows. Results offer constructive guidelines for the design of effective annular fins for heat exchangers and other thermal systems. Razelos (1983) [13] analyzed circular-sectioned pin fins whose diameters change with altitude according to a given profile. The most efficient diameter and height for a reducing over-fin volume at specific heat dissipation rates were determined in the paper. Hati and Rao [14] investigated analytically the optimal size of cooling fins dissipating heat by combined convection and radiation. They investigated the influence of fin length and thickness, as well as thermal conductance, to optimize heat transfer criteria when some material is depleted, such as in high-temperature applications where radiation cannot be ignored. Bar-Cohen and Jelinek (1985) [15] proposed an analytical model to improve the thermal configuration of longitudinal rectangular fins exposed to convection. They were looking to develop the most efficient method with the least material use for heat management. Fin superior tuning: The optimization of the fin arrangement was carried out by considering parameters such as spacing and position, which directly affect thermal performance. The theoretical thermal performance was evaluated using illustrative examples, providing insights into the design of efficient fin arrays for heat exchangers. Hrymak et al. (1985) [16] proposed a numerical optimization method to identify optimal fin geometries that account for both convective and radiative heat losses. Using an iterative computational approach, they solved the governing heat-transfer equations for variable-section fins, enabling the optimization of complex geometries beyond the scope of traditional analytical solutions. The optimization approach proposed by Sonn and Bar-Cohen (1981) [17] for the cylindrical spine, illustrated in Fig. 2, minimizes spine volume. The optimum diameter and height for a spine are given by:

$$d_o = 1.503 \left(\frac{hV^2}{k} \right)^{1/5} \tag{3}$$

$$b_o = 0.564 \left(\frac{Vk^2}{h} \right)^{1/5} \tag{4}$$

For the same geometry, Aziz A. (1992) [18] proposed an alternative optimization formula, derived based on maximizing heat dissipation, as expressed by:

$$d_o = 0.917 \left(\frac{q_b^2}{hk\theta_b} \right)^{1/3} \tag{5}$$

$$b_o = 0.440 \left(\frac{q_b k}{h^2 \theta_{b\theta}} \right)^{1/3} \tag{6}$$

Aziz A. (1992) [18] proposed optimum formulas for various spine fin geometries, as illustrated in Fig. 2, based on two criteria: minimum volume and maximum heat dissipation. The formulas are presented in two sections, corresponding to the minimum volume and maximum heat dissipation for each spine fin.

a. The optimal diameter and length of the fin based on minimum volume
   • For a conical fin

$$d_o = 1.9536 \left(\frac{hV^2}{k} \right)^{1/5} \tag{7}$$

$$b_o = 1.0008 \left(\frac{Vk^2}{h^2} \right)^{1/5} \tag{8}$$

   • For a concave parabolic spine

$$d_o = 2.0968 \left(\frac{hV^2}{k} \right)^{1/5} \tag{9}$$

$$b_o = 0.7505 \left[\frac{q\,k}{h^2 \theta_b} \right]^{\frac{1}{3}} \tag{10}$$

   • For Convex parabolic spine

$$d_o = 1.798 \left(\frac{hV^2}{k} \right)^{1/5} \tag{11}$$

$$b_o = 0.7877 \left(\frac{Vk^2}{h^2} \right)^{1/5} \tag{12}$$

a. The optimal diameter and length of the fin based on maximum heat dissipation
   • For a conical fin

$$d_o = 1.0988 \left[\frac{q^2}{hk\theta_b^2} \right]^{1/3} \tag{13}$$

$$b_o = 0.7505 \left[\frac{q\,k}{h^2 \theta_b}\right]^{\frac{1}{3}} \tag{14}$$

- For a concave parabolic spine

$$d_o = 1.1746 \left[\frac{q^2}{hk\theta_b^2}\right]^{1/3} \tag{15}$$

$$b_o = 1.0838 \left[\frac{q\,k}{h^2 \theta_b}\right]^{\frac{1}{3}} \tag{16}$$

- For Convex parabolic spine

$$d_o = 1.0262 \left[\frac{q^2}{hk\theta_b^2}\right]^{1/3} \tag{17}$$

$$b_o = 0.5951 \left[\frac{q\,k}{h^2 \theta_b}\right]^{\frac{1}{3}} \tag{18}$$
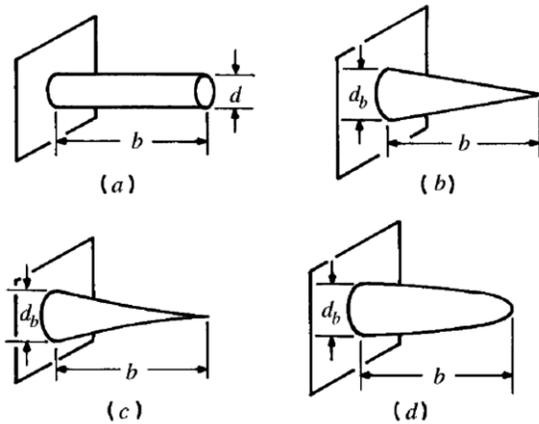

Figure (2): Illustrations of spinal geometry: (a) cylinder spine, (b) conical spine, (c) concave parabolic spine, (d) convex parabolic spine

Buccini and Soliman (1986) [19] Optimum dimensions of annular fin assemblies to enhance heat dissipation performance. The study is about stationary one-dimensional heat conduction and convection to determine the dimensions of the fin that minimize thermal resistance. These results could serve as a guideline for the design of an annular fin with improved thermal performance. Chung and Zhang (1991) [20] consider the optimal design of longitudinal fins with minimum mass, subject to the condition that radiative heat transfer between the fin and the base is accounted for. Variational principles are used to obtain the fin that provides the maximum heat dissipation per unit mass. The results provide a perspective on the effectiveness of fin design in situations where minimum material use is key. Yeh

(1995) [21] investigates heat transfer on fin arrays under both boiling and convection boundary conditions. To maximize heat transfer performance and reduce material usage, a power-law relationship between the local heat transfer coefficient and the temperature difference is used, based on Lagrange multipliers. Lalot et al. (1999) [22]. This model was developed to evaluate annular fins with two different materials, for example, a high-conductivity coating on the base. It provides formulas for the temperature distribution and fin efficiency, accounting for the coating layer thickness and its properties. Efficient conduction significantly increases fin performance by smoothing temperature gradients and facilitating the design of fins in heat exchangers and thermal systems. Such an approach will yield functional implications for heat exchangers and other thermal systems regarding the optimization of fin designs. Early studies on the optimization of fins with extended surfaces focused on how fin performance depended on the choice of material and boundary conditions. Yu and Chen (1999) [23] used an analytical optimization model for circular fins with temperature-dependent thermal conductivity and convection, yielding optimal shapes that minimize overall thermal resistance. Based on this approach, Yeh (2001) [24] analyzed rectangular fin arrays, accounting for heat conduction between fins, and demonstrated the effects of fin spacing, aspect ratio, and other geometric parameters on heat dissipation. Claudio and Franco (2001) [25] developed an analytical one-dimensional form for a rectangular longitudinal fin of finite length with different heat-transfer coefficients at the surfaces and the tip. Their optimal design focused on weight reduction subject to thermal and geometric constraints, resulting in the development of optimum charts for the Biot number (N), aspect ratio, rate of heat transfer, and fin mass, and challenging the universal validity of the Half-Thickness Rule. Following, Mokheimer (2002) [26] showed that annular fin studies under natural convection need to account for variations in the heat-transfer coefficient throughout, pointing out that constant values would result in erroneous predictions of efficiency. In a similar thermal–mass transfer cell, Kundu (2002) [27] investigated straight tapered fins operating in wet conditions and derived analytical solutions that accounted for condensation effects. Optimization with Lagrange multipliers demonstrated that wet fins need to be longer than dry fins for optimal performance, and a volume comparison between fins was used to assess the appropriateness of fin geometry

for combined cooling dehumidification duties. For instance, Razelos and Krikkis [28] also study the optimal heat dissipation of individual rectangular-circular longitudinal fins. It makes an analysis based on conduction and convection heat transfer, with special attention to fin geometry and material. The paper develops optimal fin length and thickness based on thermal performance while minimizing material use. The results provide us with applicable rules of thumb for designing effective fins across different heat transfer applications. Bejan and Kraus (2003) [29] provide an overview of the optimal dimensions of longitudinal fins under convective and radiative heat transfer. The fin optimizing formulation is presented using a variational method to obtain optimal fin dimensions that minimize thermal resistance and maximize heat dissipation efficiency. The parameters, such as the fin length, thickness, and spacing, are optimized with due consideration of convection and radiation. The results provide design recommendations for finned surface geometries in heat transfer applications where both conduction and convection are significant, such as in electronic cooling devices and heat exchangers. Correlation equations for the design of a uniform-thickness annular fin were also developed by Arslanturk (2005) [30]. By setting a fin volume, the optimal geometrical parameters for heat transfer are obtained. The critical radius ratio depends on the Bi number and the fin volume. These help engineers design better fins to dissipate heat. Naphon (2006) [31] addressed heat transfer and efficiency of annular fins under dry, partially wet, and thoroughly wet conditions, wherein axial conduction in the fin body was considered using the conservation equation as solved by the central finite differencing method to obtain the temperature distribution. The surface effect on heat transfer was significant, with a thoroughly wet condition providing the most efficient heat transfer. These results will facilitate the optimal design of the pin-fin heat sink for electronic cooling. Chen et al. (2007) [32] solve an inverse heat transfer problem to determine the space- and time-dependent convection heat transfer coefficient of an annular fin. Using a conjugate gradient-based inverse method, the work demonstrates that if the history of temperature at certain fin positions is available, then the heat transfer coefficient for convection can be predicted without explicitly studying its form. The accuracy of the inverse analysis is confirmed through simulations of the temperature field, which provide excellent predictions of the heat

transfer coefficient, temperature fields, and thermal stresses in different cases. Sharqawy and Zubair (2007) [33] proposed an analytical model to study the performance and optimal dimensions of annular fins under heat and mass transfer conditions. The closed-form solutions for the temperature distribution and efficiency are obtained under fully wet conditions for both sensible and latent transfers. The results are quite useful for the design and optimization of heat exchangers for condensation processes. Chen et al. (2007) [34] proposed an inverse heat transfer approach to determine the heat transfer coefficient and fin efficiency of annular fins of a nested-tube heat exchanger due to natural convection. The fin was partitioned into sub-domains and fitted with the experimental points by means of finite difference and least squares approximations. Results indicate that heat transfer increased while fin efficiency decreased as the fin spacing increased, providing a guideline for optimizing fin design. Yang et al. (2007) [35] investigated the thermal and hydraulic effectiveness of p-fin heat sinks with circular, elliptical, and square cross-sections, testing 12 configurations in-line and staggered. In downstream configurations, they observed that circular fins were much more sensitive than those used in inline planes and were generally insensitive to fin spacing. Staggered fin arrangements increased heat transfer with fin density across all configurations. The elliptical fins showed the minimum pressure drops and thermal resistance for a given surface area and pumping power combination. Kou et al. (2008) [36] studied the thermal performance of heat sinks with longitudinal fins in rectangular, square, and cylindrical shapes to optimize cooling effectiveness while minimizing material consumption. They include the Bi number, heat transfer coefficient ratio, and shape factor in the analysis, yielding an implicit relation for heat transfer that is numerically solved to determine an optimum fin length and thickness. The impact of fin shape on thermal resistance is discussed, and valuable information is provided to help design an effective heat sink. Copiello and Fabbri (2009) [37] published a work on the optimization of longitudinal wavy fins' performance by means of an MOGA. The goal is to increase heat transfer and reduce hydraulic drag. The Non-dominated Sorting Genetic Algorithm II (NSGA-II) is employed in the study to analyse trade-offs between these conflicting objectives. The incident demonstrates that some fin designs can achieve remarkable reductions in hydraulic resistance with

little compromise in heat transfer capability. These results have significant implications for the optimization of efficient finned heat exchangers. A correlation for annular fins, but an important point in this work is that it considers the variation in thermal conductivity with temperature. The problem is analyzed using the nonlinear fin equation with variable thermal conductivity and is analytically solved using the Adomian decomposition method. The optimum radius ratio for maximum heat transfer depends on the Bi number and the fin volume relative to the change in thermal conductivity. The optimum geometric parameters for heat transfer are determined while the fin volume is held constant. The correlation equations are applicable over the range of Bi numbers and fin volumes, allowing thermal design engineers to use these results effectively in practical optimization of annular fins with temperature-dependent material properties. Azarkish et al. (2010) [39] also developed an analytical model of the optimal longitudinal fin arrangement that accounts for heat convection, mass transfer, and other factors. It enhances heat-transfer rates and guides improvements in thermal performance. Nagarani et al. (2012) [40] investigate the performance optimization of EAFs using a GA. Such optimization accounts for key parameters, including the fins' major and minor dimensions, as well as the thermal conductivity and the convection heat-transfer coefficient. The purpose of the (GA) is to optimize heatsink efficiency while minimizing material cost. The results reveal that the preferred fin design significantly enhances thermal performance compared with circular fins. This approach yields a systematic, efficient pin-fin design for mini- and microchannel heat exchangers.

Hamadneh et al. [41]. The Particle Swarm Optimisation (PSO) technique for optimising pin-fin geometry to enhance thermal and fluid performance was adopted by Maiti et al. Entropy Generation Minimization was applied to optimize thermal resistance and pressure drop in a heat sink. The fin profile was found to depend on the aspect ratio, the axis ratio, and the Reynolds number. Elliptic pin fins had higher heat transfer and higher fluid friction. These observations assist in optimizing pin-fin geometry for heat transfer enhancement. Huang and Chung (2014) [42] used an iterative conjugate-gradient method to identify the optimal thoroughly wet annular fin arrangement for maximum heat effectiveness. Their analysis included both

temperature-dependent and Bi numbers, providing invaluable information for the design of fins operating under moist conditions to improve heat transfer. The primary objective was to determine which shapes are most efficient in inducing these effects. Kundu et al. (2015) [43] introduced an analytic theory for the design of annular step fins. They reduce the temperature field to exponential forms, enabling engineers to design fins that maximize heat transfer and enhance thermal performance. Nguyen and Yang (2016) [44] propose a modified Newton–Raphson-based design to minimize the volume of nonlinear longitudinal cooling fins. The fin profile is modelled as a B-spline curve whose control points are optimization variables, and the profile is modified via a volume-updating procedure to minimize volume for a given heat dissipation. The method is verified in four cases with different boundary conditions and thermal properties; it proves to be an effective and accurate parameter for fin optimization design. Senapati et al. (2016) [45] studied natural convection heat transfer from a horizontal cylinder with annular fins experimentally and using CFD simulations. They investigated the effects of fin spacing, height, and Rayleigh number on heat transfer and observed that fins effectively promote the heat dissipation relative to finless cylinders. The optimal fin configurations enhanced heat-transfer effectiveness and, consequently, reduced material consumption, providing a valuable basis for heat exchanger and thermal system design. Iqbal et al. (2017) [46] reported that a genetic algorithm and a finite element method optimized finned annuli with triangular fins under laminar flow. They intended to achieve high heat transfer, low pressure loss , and low cost. Optimum configurations have at least 18 fins and achieve cost-effective solutions while reducing pressure loss. Hajmohammadi (2018) [47] optimizes the design of a multi-scale annular fin on a pin fin for maximum heat transfer with minimum energy and cost. The "Optical metagene genetic algorithm" constructs the theory and also performs geometric-constrained optimization using a genetic algorithm. Geometric variables improve thermal performance but make optimization difficult -- these design rules guide the design of cooling devices. Sa et al. (2018) [48] studied the thermal hydraulic performance of gapped circular pin-fins in a cooling channel through three-dimensional Navier–Stokes equations and low-Re shear-stress turbulence model. Their work was to investigate the effect of geometric shape on heat

transfer and pressure drop to maximize heat-exchange performance by minimizing pressure loss, with a focus on the Nusselt number and pressure loss coefficient. It was found that Slot(K)-LM placement of the front and rear gaps has a significant effect on performance: the corresponding optimal configurations enhance thermal efficiency and decrease hydraulic resistance. The study provides guidance for effective pin-fin cooling design. Al Doori (2019) [49] used CFD simulations to evaluate the influence of fin spacing, tube shape, and Re number on convection heat transfer and pressure drop in annular-finned channel heat exchangers with circular, elliptical, and flat tubes. It was found that the elliptical and flat tubes enhanced heat transfer while increasing the pressure drop; this indicates, however, that a compromise between thermal and hydraulic aspects should be made when designing efficient heat exchangers by tuning tube geometry and fin arrangements. Ahmad et al. (2020) [50] present a neuroevolutionary model based on the combination of a neural network and a genetic algorithm. This technique predicts the temperature distributions in fin geometries to enhance heat transfer performance. It simulates complex thermal performance, facilitating the design of heat transfer systems. It is suggested that this method can make a useful contribution to thermal system design by using more realistic temperature profiles. Kim (2021) [51] developed a model to analyze the thermal performance of finned tube annuli with different fin geometries. It was shown that, for air-cooled grey-body annuli, efficiency increases from straight fins to CSF and other designs. It can provide references for the optimal design of fins in heat exchangers. Sertkaya et al. (2021) [52] studied the effect of pin fin height (H), spacing (S), and orientation on the natural convection heat transfer characteristics in inline plate-and-pin-fin heat sinks. Improved fin height, reduced spacing, and optimized fin angle enhance thermal performance when designing the heat sink for electronic cooling. Sarwe and Kulkarni (2022) [53] used the DTM to investigate heat transfer in annular fins of variable thermal conductivity. They concluded that DTM can estimate temperature distributions and fin efficiencies quite successfully, even for temperature-dependent conductivity. These results, verified using analytical and finite-difference approaches, demonstrate the accuracy and validity of the DTM with confidence. Tahrour et al. (2023) [54] have conducted a CFD study on annular finned-tube: concentric, eccentric, perforated, serrated, and star-shaped. According to the results, the mass performance of the star-shaped fin (S-SFT) is superior to that of other fins, and the eccentric circular fin (ECFT) shows a balance between heat transfer rate and pressure loss, providing useful information for engineering perspectives on improving heat exchangers. Yang et al. (2024) [55] discussed an ATEG with a pin-fin design for automotive waste heat recovery and studied its thermoelectric and thermo-mechanical performance based on 3D model calibration. They optimized five parameters using the Taguchi L25 orthogonal array: exhaust temperature, mass flow rate, fin height, fin diameter, and the number of fins. The best setting was 34.11 W, which is 18.7% higher than the baseline. The pin fins were better than the plate fins, with increases of 5.83% and 4.82% in power and stress, respectively. Such results offer instructive suggestions for the optimal design of ATEGs to improve back extraction energy from vehicles. Bhowmick et al. (2024) [56] used ANSYS Workbench 2021 R2 to investigate thermal analysis of three types of fins, NACA 2412, tapered triangular, and pinned fin in free convection. They aimed to use five materials: brass, aluminum, copper, steel, and iron. The results show that aluminum airfoil fins provide better heat dissipation, followed by copper tapered fins, and brass-pinned fins were found to be the worst. These findings are encouraging for the design of fin shapes and materials to enhance heat sink efficiency. efficiency. Han et al. (2024) [57] employed simulation and RSM to optimize the annular, fin phase change heat storage elements. They also discovered that the internal pipe radius, fin thickness, and fin length have a significant impact on capacity. The best F1 had a radius of 12.823 mm, a thickness of 10 mm, a length of 10 mm, and an energy storage capacity of 1647.548 kJ. These findings provide a systematic route to enhance heat storage systems. Ali (2024) [58] carried out an experimental investigation to assess the heat transfer behavior of copper horizontal tubes for integral and pin-fin shapes during the free convection condensation of steam. The performance characteristics showed that both finned tubes provided improved heat transfer compared to the plain tube. In particular, 35.77% and 32.96% enhancements in heat transfer performance were observed with the pin-fin tube and the integral fin tube, respectively. The pin-fin had a \~4% higher effectiveness than the integral fin, which was ascribed to less flooding phenomena. These results indicate that by appropriately shaping the fins, excellent condensing heat transfer performance can be achieved.

Ali et al. (2025) [59] studied the natural convection heat transfer of perforated annular fins on a hot vertical cylinder using ANSYS Fluent. They studied three Rayleigh numbers (Ra = 0.68×10^7, 1.37×10^7, and 1.8×10^7) and fins with fin pitch-to-diamter ratio (S/d) of mid ranging from s/d =2 to 5 including tail fin (s/d=5.8). It is found that this number increases with Ra (up to for S/d = 2 and at least for S/d = 5.84). The improved heat transfer by natural convection is accentuated by the fin perforation and spacing. Maji et al. [60] applied CFD simulations and swarm intelligence algorithms to develop a perforated fin design for a heat sink. The results indicated that perforated fins exhibit more effective heat transfer performance than solid fins, particularly at higher perforation rates. The optimizations show effective perforation configurations, and the improvement in thermal performance is considerable. These findings could provide useful references for promoting heat sink performance in electronic cooling. Oderinu et al. [61] studied the distribution of temperature and stress in an annular fin, with a temperature-dependent emissivity based on a power law. They derived an energy balance with varying emissivity and solved the problem by minimizing partition, which led to polynomial solutions. The influence of thermo-physical properties on fin temperature and stress was observed to be more pronounced, underscoring the noticeable variation in emissivity in the design of fins. Their result matched well with those in the literature, which verified the validity of the method.

Fin technology has similarly evolved over the past couple of centuries, transforming thermal design from an art into a precision science. Early stepping stones, such as Ingenhousz's wax-covered rods and Harper and Brown's analytical framework for conduction and convection, laid the foundations of fin theory. Interactions among geometry, material choice, and multiplex heat transfer have been scrutinized over the years, leading to numerous optimization procedures that find application across a wide range of industrial fields. The reviewed works show consistent performance trends: optimized geometries consistently achieve 2040% higher heat-transfer performance than conventional models, and material-efficient profiles can reduce material use by 1530% without compromising performance. Technologies such as perforated fins, gapped fin pins, elliptical shapes, and airfoil cross-sections have shown enhanced thermal-hydraulic performance, with reduced pressure drop and increased Nusselt number.

In addition, models that account for radiation, wet-surface operation, and variable thermal properties have enhanced predictability, thereby enabling engineers to develop solutions for high-temperature, condensation, and/or natural-convection regimes. Contemporary methods further employ computational fluid dynamics, inverse heat transfer analysis, and metaheuristic optimization algorithms, such as genetic algorithms and particle swarm optimization, to efficiently search large design spaces. These methods, experimentally verified, have yielded optimized configurations that maximize dissipation per unit mass and minimize resistance with respect to fin spacing, while accounting for the trade-off between heat-transfer performance and manufacturability. Additive manufacturing is a disruptive enabler, enabling the construction of complex, topology-optimized fin geometries that would otherwise be infeasible. In particular, hybrid concepts combining longitudinal, annular, and pin-fin designs show promising performance, particularly under stringent compactness requirements and high flexibility. From electronics cooling to automotive waste-heat recovery, phase-change storage, and aerospace systems, the trend is clear: fin design is evolving toward innovative, adaptive, and embedded thermal solutions. Next-generation advances will likely integrate innovative materials with in situ sensing and control to yield fins that optimize themselves in real time. Designs will be guided by AI-based multi-objective optimization to unknown combinations of thermal efficiency, hydraulic performance, and sustainability. The aggregate evidence highlights a radical departure from evolutionary-geometry-based enhancements toward an integrated design approach that brings together physics-based modeling, computational intelligence, and advanced manufacturing. Fins in this regime are no longer passive features; they become engineered thermal interfaces that shape the energy landscape of future technologies.

Nomenclature

b_o optimum fin length, m

d_o optimum fin diameter, m

h convective heat transfer coefficient, W/m^2-K

k thermal conductivity of fluid, W/m-K

n indicates the type of fin profile

q heat flux, W/m^2

x length coordinate, m

x_b fin length, m

y fin thickness function, m

y_b fin thickness at the base, m

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