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Free Vibration Behavior of Cantilever Plate with Localized Stiffness Loss at the Clamped End

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A B S T R A C T

Cracks and discontinuities present in the structure have a significant influence on the vibration properties of a component. The research work presented in this paper investigates the effect of a crack present at the fixed edge of the cantilever steel plate on the vibration frequency and mode of the steel plate. The analysis presented above adopts a combination of modeling techniques based on CPT and the numerical analysis technique based on FEM in the ANSYS APDL package and attempts to establish the amount of variation in the vibration properties of a plate with the rise of the a/W ratio of the crack. A detailed finite element analysis has been conducted with special attention to the modeling near the cracked area. The natural frequencies and modes have been analyzed for various values of the crack ratio and validated with theoretical values based on the frequency equations with an account for flexibility caused by the presence of cracks. Both theoretical and FE results show that with an increase in the value of the crack ratio, natural frequencies tend to reduce, especially for mode 1. Mode shapes show localization near the crack area, which indicates the effect of the crack on the system stiffness and vibration. The proposed equations derived by fitting the numerical results into the classical frequency equations offer a reliable means for determining the effect of the crack on the frequency shift without having to go to experimental methods. This research highlights the capabilities of a combined analytical and numerical approach to the prediction of the vibration response of cracked systems with a view to developing the area for complex damage configurations.

1. Introduction

Cantilever plate is a basic form of structural components which finds widespread use in different branches of engineering like aerospace engineering, design of various mechanical components, and micro-electromechanical systems. The cantilever plate with its simply supported form and high sensitivity to structural defects can be considered an optimal model for studying the characteristics of various vibrating systems with different boundary conditions and defects. It is common knowledge that the studied plates are typically subjected to dynamic loading during their operating processes.

The most fundamental and common type of discontinuity is that with a crack, which originates from fatigue, a production imperfection, and/or a heavy load applied to the structure. In some particular cases, a crack located in the proximity of the fixed support of a cantilever system produces a local flexibility variation with the potential to generate a remarkable influence on the vibration characteristics of a structure. If such defects are not detected in time, it would lead to a disastrous consequence. Hence, vibration analysis of cracked plates finds its importance for structural health monitoring and reliability analysis.

The theoretical analysis part, for instance, witnessed the emergence of a number of analytical theories for analyzing the dynamics of plates. The point is that all these analytical theories were established based on the premise of ideal geometry and perfect boundary conditions. The phenomenon brought about by the complexity of irregular geometry and non-standard boundary conditions—that may be faulty or imperfect at the boundaries of a clamped or fixed boundary—remains such that analytical theories are ineffective for modeling plates with discontinuities or imperfect supports.

Some authors have investigated the effects of the crack's existence on vibration characteristics of structural members such as a beam and a plate with different boundary constraints. The first crack model in a structure based on flexibility was introduced by Dimarogonas for the first time [1]. Later, a crack model for a beam based on the shear deformation theory was established by authors Christides and Barr [2]. The modal analysis for a cracked plate was conducted through FEM by authors Mia et al. [3]. These experimental studies, as evidenced by Cawley and Adams [4], proved these frequencies can be sensitive damage indices, particularly for a cantilever configuration with a crack located within the vicinity of a zone of high moments of bending. Other works have used more current methodologies, incorporating experimental validation. For example, Su & Ye [5] used a vibration-based damage identification technique to explore the problem of cracked composite plates, while Agarwalla & Parhi [6] analyzed the effect of the angle and length of a crack extending to the mode shapes. Saadatmorad & Jafari [7] developed a method of localizing a crack by curvature of mode shapes and wavelet analysis, while the efficacy of this method in accurate localization of a small crack with the aid of vibration signals was verified. The authors indicated the sensitivity of mode shape curvatures to changes of stiffness to ensure the suitability of vibration signals for methods of damage identification. Malekzadeh & Beni [8] studied the vibration of a cracked functionally graded plate with through cracks by means of the generalized differential quadrature method. Its conclusion emphasized the substantial decrease in natural frequency owing to the influence of cracks, proving the efficiency of the numerical solution in modeling the dynamic behavior. Eshete et al. [9] conducted both numerical and experimental work on GFRP composite beams, concluding the substantial influence of natural frequency owing to the rise in the length of the cracks. Chen et al. [10] obtained the closed form of the exact solution for the free dynamic vibration of the rectangular plates with edge cracks, which can be regarded as the theoretical approach for validation. Shehab et al. [11] and Adimas et al. [12] further extended the work for functionally graded composite plates and hybrid composite plates, respectively. The influence of crack propagation on vibration responses for cantilever plates has been investigated by Xiong et al. [13]. Analytical studies by Imam and Kumar [14] in cracked plates and beams have been carried out to emphasize numerical modeling for damage analysis of structures. An analytical model for an orthotropic plate that has partially restrained boundaries has been proposed by Zhang et al. [15]. Das et al. [16] developed an integrated numerical and experimental design based on ideas from the frequency domain to facilitate the process of identifying cracks in laminated composite beams. One of the early contributions, although not so recent but still forming the theoretical background of the majority of the latest studies around the multimodal cracks of the cantilever beam composed of composites, is the one of Murat and Kisa [17]. Rezaee etal. [18] have developed a novel form of the disturbance function method and provided a novel point of view regarding the modeling of open edge cracks of the beam problem using the dynamic solution method schemes; all the above studies stressing the fact that the loss of rigidity affected by the free edge has a profoundly negative impact on the dynamics and should receive much more attention to obtain reliable models regarding the prediction of such phenomena.

In this research work, a detailed analysis has been made to assess the effect of a single edge crack, which has been created on the clamped end of a cantilever steel plate, on the natural frequencies and modes of vibration of the steel plate. The analysis is carried out by applying a free vibration technique using analytical and numerical techniques in addition to the development of an empirical formula. In the numerical technique, a finite element analysis has been made by using the ANSYS APDL software to analyze the steel plate and calculate its natural frequencies and modes of vibration. A set of values for the ratio of the length of the crack to the width of the steel plate (a/W = 0.1 to 0.5) has been chosen to qualify the effect of the size of the created crack on the mode of vibration.

2. Theoretical Background

Despite these developments, the requirement for empirical expressions to describe the computation of the effects of the crack on the vibrational phenomenon for the clamped-end is still required owing to the fact that the analytical solution for the computation of the vibrational phenomenon is inadequate on the crack locations for the clamped-end.

In fact, it can justly be stated that an essential element, or aspect, of either Mechanical Engineering or Structural Engineering encompasses an analysis evaluation with regard to several dynamics contained inside said plate and beam elements. The model system with regard to said cantilever support for said fixed-free plate very likely has relevance with regard to model systems as they relate to analyzing the significance of parameters contained inside said model system, as said parameters are affected due to said dysfunctional system from said perspective of said discontinuity.

Free vibration of intact benchmark is mainly described through classical plate theory-CPT theory, also called Kirchhoff plate theory. The vibration of a thin isotropic, homogeneous plate subjected to transverse vibration can be modelled as in equation (1) [19] [20]:

$$D\nabla^4 w(x,y,t) + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} = 0 \qquad (1)$$

Where $D = \frac{Eh^3}{12(1-v^2)}$ is the flexural rigidity, ρ is the mass density, h is the plate thickness, $w(x,y,t)$ is the transverse deflection, E is Young's modulus, and v is Poisson's ratio. For cantilever boundary conditions, appropriate constraints are imposed at the clamped edge, while the free edge remains unconstrained. Then, the natural frequencies of cantilever plate can be calculated from equation (2):

$$\omega_{mn} = \frac{\beta_m^2}{L^2}\sqrt{\frac{D}{\rho h}} \qquad (2)$$

Where β_m equal to ($\beta_1 = 1.875, \beta_2 = 4.694, \beta_3 = 7.855, \ldots$)

The introduction of a crack, especially at or near the fixed support, Figure (1), introduces a local discontinuity in the stiffness matrix, then reduce the flexural rigidity D. The disrupts the strain energy distribution, particularly in the vicinity of the crack tip, where stress concentrations occur. As a result, the vibrational energy is redistributed, often leading to a reduction in natural frequencies, especially for lower modes where global stiffness plays a dominant role. This behavior becomes more pronounced as the crack length increases relative to the plate width. Several modeling techniques have been proposed to represent cracks in vibrational analyses. Analytical models, such as those based on fracture mechanics or equivalent spring models, offer closed-form solutions but are limited to simple geometries and boundary conditions. But, for simplicity, an empirical formula can be used as given in equation (3):

$$D_{cracked} = D \times (1 - f(a)) \qquad (3)$$

Where $f(a)$ represent the reduction in flexural rigidity due to crack length (a), and equal to:

$$f(a) = \alpha \left(\frac{a}{b}\right)^{\gamma} \qquad (4)$$

Where α and γ are empirical constants determined by finite element calibration. The use of power-law form in the empirical reduction function is justified by the inherently nonlinear effect of cracks on the structural stiffness and, consequently, on the natural frequencies. As the crack length increases, the local stiffness degradation intensifies in a nonlinear manner due to stress concentration effects. This behavior is well captured by the exponent γ, which provides the necessary curvature to reflect the observed frequency decay in finite element simulations.

Then the natural frequencies can be determined from equation (5) given as:

$$\omega_{mn} = \frac{\beta_m^2}{L^2}\sqrt{\frac{D_{cracked}}{\rho h}} = \frac{\beta_m^2}{L^2}\sqrt{\frac{D \times (1 - f(a))}{\rho h}} \qquad (5)$$

In contract, numerical methods such as finite element method (FEM) allow for greater flexibility and accuracy in modeling complex crack configurations and boundary conditions. In FEM-based models, the presence of a crack is typically simulated by introducing discontinuities or by refining the mesh near the crack location to capture local gradients and deformation accurately.

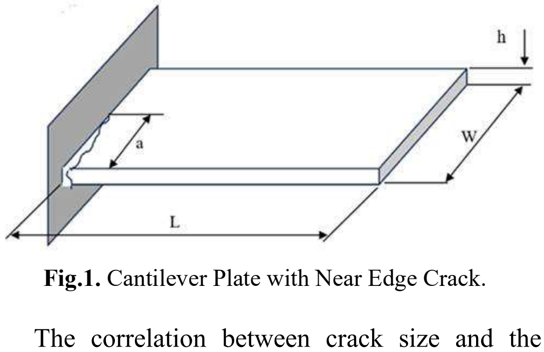

Fig.1. Cantilever Plate with Near Edge Crack.

The correlation between crack size and the percentage reduction in natural frequencies serves as a reliable indicator for damage detection and localization. Previous studies have shown that the fundamental frequency is most sensitive to damage, while higher modes may exhibit localized vibrations depending on the proximity of modal nodes and antinodes to the crack region.

1.1. Empirical Constant (α and γ) Identification via FE Calibration

To determine the empirical constants (α and γ) in the frequency reduction function, a numerical calibration process was performed using the finite element results obtained for different crack length-to-width ratios a/W. The reduction in the first natural frequency due to the presence of an edge crack at the clamped end was quantified relative to the intact plate using the normalized ratio ω_c/ω_a, where ω_c and ω_a represent the natural frequencies of the intact and cracked plates, respectively. The frequency degradation function was defined as:

$$f(a) = 1 - \left(\frac{\omega_c}{\omega_a}\right)^2 \quad (6)$$

Substituting into the empirical model (equation (4)), taking the natural logarithm of both sides yields a linearized form:

$$ln\big(f(a)\big) = ln(\alpha) + \gamma ln\left(\frac{a}{b}\right) \quad (7)$$

A linear regression was then performed on the transformed data, using the computed values of $f(a)$ and the corresponding $ln(a/b)$ values. In a similar manner, through the use of the slope, one can use the value for γ, and through the use of the intercept, $ln(\alpha)$ can be obtained, from which α can be derived through exponentiation. This offers a direct approach to the fitting process for the empirical constants using the actual activity value, thus improving the validity of the constructed frequency reduction model.

It is important to note further that the coefficients obtained are mode and dependent since all modes have the capability of responding uniquely to the local stiffness modifications that are created by the existence of cracks. In the study, the calibration process was done primarily based on the first mode, w, as it is the one that is always influenced the most because of the high curvature at the boundaries of the clamped ends. Even though the same equation can be applied to the other modes, it is advisable that the calibration process should be done for each mode separately if the accuracy of the frequency needs to be achieved.

To test for accuracy and reliability, the coefficient of determination, R^2, was used for the derived models. The coefficient of determination is a statistical measure that states variations in the dependent variable $f(a)$ are explained by variations in the independent variable a/b. Perfection in the model is given by the amount R^2= 1. In this study, the value of the coefficient for all R^2 values in the models derived for the first six modes of vibration is above 0.99, hence meaning that there is a high accuracy in the empirical model formulation as well as the resulting values derived using the Finite Element Methods. Actually, the high accuracy in the derived models along with the high value in the coefficient of determination, R^2, can be adequately supported since the constants employed in deriving the models are dependent on the discrete values given by the Finite Element Methods, meaning that the models can never go wrong in their interpolation points, hence meaning that the models are highly accurate.

In order to facilitate understanding, the empirical reduction function has been shown as $f(a) = \alpha(a/b)\gamma$. The fitting process involved six data points based on two-dimensional finite element calculations for various ratios a/W between 0.0 and 0.5. The regression coefficients, as well as R^2, are provided in Table (3). The residuals were checked to be small, and the approximation achieved R^2 values higher than 0.99, ensuring a tight link between calculations and empirical models. The model intended for application on isotropic plates, it is pertinent to state that one should proceed with extreme caution while specifying it for other geometries.

For the improvement of the accuracy of the empirical model, the values of the parameters in the empirical model (α & γ) were determined with the aid of accurate Finite Element Models for six cases with varying values of the ratio of the depth of the crack to the depth of the beam ($a/W = 0.0$ to $a/W = 0.5$). The R^2 values of the six modes were remarkably high, with values of > 0.99, which indicates the accuracy of the representation of the empirical model of the stiffness change influenced by the effect of the crack.

3. Numerical Analysis

In an endeavor to stretch the efforts accomplished within the calibrated empirical formula and to add further insights to the modal attributes of the cantilever plates with transverse cracks, it has been decided to conduct a numerical study by utilizing the finite element solution available in ANSYS APDL.

The chief purpose for accomplishing this numerical study has been to contrast the outcomes accomplished by observation related to natural frequencies and that of the calibrated empirical formula.

• The rectangular steel cantilever plate can be modeled by:

- Length (L) = 0.3 m

- Width (W) = 0.1 m
- Thickness (h) = 0.002 m
- Young's modulus (E) = 200 GPa
- Poisson's ration (v) = 0.3
- Density (ρ) = 7830 kg/m^3

These properties are inline with that of a homogeneous, isotropic material, which in general can be analyzed in relation to vibrations of structure studies. In 3D analysis, Shell181 elements have been preferred to model the plate, which represents a thin to moderately thick structure that assumes capability to model both membrane behaviors as well as bending behaviors. Shell181 elements were preferred to model the plate since their capability to model the membrane behaviors in plane as well as bending behaviors out of plane are generally quite important and required to model the thin structure experiencing stiffness loss. Additionally, in the local area around the crack tip, it has been assumed to model the mesh of dense area to represent the sharp variations of stresses and strains. This represents balancing of effectiveness of stiffness loss modeling with computational requirement. Modeling of the crack present in the structure of the thin plate using Shell181 elements has been adopted in earlier researches by Kisa et al. [3] and Malekzadeh & Beni [8] among others. In the simulation, one side of the plate was fully clamped, and the zero displacement as well as rotation was fixed for all DOF through the DDAM simulation, defining the full clamping of the cantilever plate simulation process. Finally, regarding the final simulation process of the crack force simulation, a through thickness edge notch was incorporated, involving the elimination of elements through thickness for the domain where the simulation process needs to be done. Lastly, the mass influenced by the simulation process is also considered to be less than 1% of the simulation process.

This further verifies that there was no impact assessment identified regarding the mass but identified regarding the stiffening impact as might have been caused through the creation of a possible crack. The description in the Numerical Analysis section has been expanded accordingly. The crack length was varied across six values:

$$a = \{0.00, 0.01, 0.02, 0.03, 0.04, 0.05\} \rightarrow \frac{a}{W} = \{0.0, 0.1, 0.2, 0.3, 0.4, 0.5\} \quad (8)$$

In all possible scenarios, a structured mesh is employed coupled with the element size control mechanism such that smaller-sized elements were distributed in and about the region with the existing crack. The convergence test for the mesh is performed to ensure that the criterion is met where the natural frequencies are independent of the mesh size. But for the density value in the mesh, it relies on the costs associated with the computation processes coupled with the criterion for which the modal shapes about the crack region must be identifiable. The mesh was very fine around the tip of the crack. Also, the standard quadrilateral elements were used with the size of the element being typically (0.5 mm) around the tip of the crack. The singular elements were not used because the main attention of the current problem is towards the global vibration response.

A modal analysis of free vibration was performed with the Block Lanczos algorithm, a robust algorithm with high efficiency in computing multiple eigenvalues. The first six natural frequencies with their modes were determined for every configuration of the cracks.

It is noticed that there is a gradual reduction of natural frequencies with the increase of the length of the crack, but the reduction is more significant in the lower modes. Table (1) below shows the numerical values of the natural frequencies of the six modes at the different a/W ratios. The major reduction was noticed in the fundamental mode, which reduced by 10.4% at $a/W = 0.5$. But the higher modes were reduced to a smaller extent. Moreover, the distortion of the mode shape was noticed with the increase of the length of the crack.

These trends validate the high sensitivity of the dynamic response to the severity of the crack and confirm the correctness of using frequency parameters for crack identification. The calibrated expression was also found to match well with the numerical predictions, which confirms the validity of the FEM model established for simulation.

4. Results and Discussion

These trends have verified the sensitivity of the dynamic response to the severity of the crack as well as the suitability of using frequency parameters in identifying the severity of the crack. The expression calibrated has also been found in good agreement with the numerical results, thereby validating the FEM model developed for simulation.

Table (1) and figure (3) provide a summary of the first six natural modes for various a/W ratios. In the intact plate, when $a/W = 0.0$, for mode one, a frequency of 18.489 Hz was observed, whereas for mode six, it was 638.08 Hz. On increasing the crack length, it was noticed that there was a systematic decrease in the values of in for all modes. For a/w = 0.5, the values decreased to 16.570 Hz for mode one, showing a decrease of 10.4%, while for mode six, it was 480.96 Hz, which showed a decrease of 24.6%.

Table 1. First Six Natural Frequencies Reduction with a/W Increasing.

Reduction in First Six Natural Frequencies (Hz)						
M	a/W					
	0.0	0.1	0.2	0.3	0.4	0.5
1	18.489	18.407	18.172	17.790	17.258	16.570
2	113.83	112.88	110.32	106.60	101.85	96.163
3	115.51	115.12	114.60	113.99	113.15	112.02
4	324.50	322.73	318.02	309.74	297.04	279.89
5	357.50	355.01	350.58	345.84	341.47	337.40
6	638.08	633.01	617.00	591.54	556.00	480.96

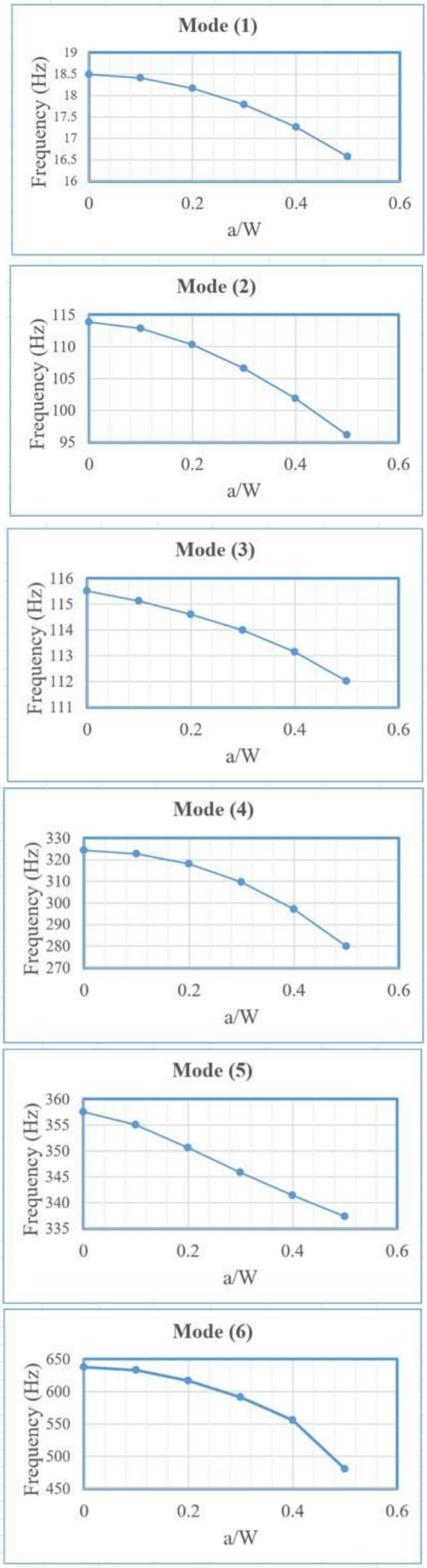

Fig. 3. First Six Natural Frequencies Reduction with a/W Increasing.

This behaviour is attributed to the localized loss of stiffness near the clamped edge, which sees the highest bending moment in a cantilever configuration. The presence of the crack significantly reduces the flexural rigidity in this region, thereby lowering the ability of the system to resist deformation under vibratory excitation.

The above Table (2) and Figure (4) show the percentage decrease in natural frequencies with respect to the values of an intact plate. The first mode is found to be very sensitive to the growth of crack with respect to percentage decrease that varies from 0.44% to 10.36% with an increase in crack ratio from 0.1 to 0.5 for a particular value of $a/w = 0.5$. Likewise, a decrease is observed in the second and fourth modes with respect to percentage decrease values of 15.52% and 13.75% for the highest value of crack ratio. It can be found that the third and fifth modes are found to be more insensitive with respect to percentage decrease values of 3.02% and 5.62%, which clearly reveal that the associated modal deflection shapes are less sensitive to the location of the crack.

This differential sensitivity points out the role of modal strain energy distribution; namely, modes that have significant curvature or deformation near the crack location show more degradation in frequency. The third mode might have a nodal line close to the crack and hence will undergo minimum displacement, and thus is not as much influenced by the local change in stiffness.

Figure 5(a) presents the variation of the mode shapes with respect to increased a/W. From mere visual inspection, it can be easily identified that the modal deformation patterns are distorted. Especially in the lower modes. For $a/W = 0.0$ (intact plate) mode shapes present regular symmetric patterns expected for a homogeneous cantilever. When, the crack length is increased, asymmetry and softening become apparent in the mode shapes near the clamped boundary due to localized compliance and changed boundary stiffness.

The spatial change in mode shapes is especially relevant for vibration-based damage identification, as it provides direct visual evidence of crack-induced changes.

Table 2. Percentage Reduction of First Six Natural Frequencies with a/W Increasing.

Percentage Reduction in First Six Natural Frequencies (%)						
M	a/W					
	0.0	0.1	0.2	0.3	0.4	0.5
1	0	0.444	1.715	3.781	6.658	10.379
2	0	0.835	3.084	6.352	10.524	15.521
3	0	0.338	0.788	1.316	2.043	3.021
4	0	0.545	1.997	4.549	8.462	13.747
5	0	0.697	1.936	3.262	4.484	5.622
6	0	0.795	3.304	7.294	12.864	24.624

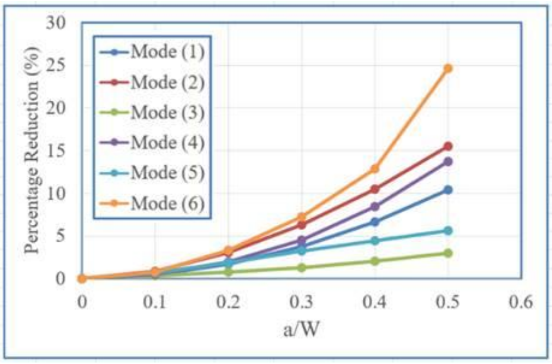

Fig. 4.: Percentage Reduction of First Six Natural Frequencies with a/W Increasing.

The shift in curvature and displacement intensity along the plate surface reinforces the quantitative frequency analysis and supports the use of combined modal metrics for crack detection.

The comparison between FEM simulation and empirical results shows strong agreement across all crack configurations. The deviation between the two approaches remains within acceptable bounds (<5% for most modes), affirming the accuracy of the modeling assumptions, meshing strategy, and boundary condition implementation in the numerical model. Such correlation enhances the credibility of both the empirical formula and numerical frameworks and establishes a reliable foundation for future applications involving damage quantification and predictive modeling.

The findings of this study provide valuate insights into the vibrational degradation mechanisms in cracked cantilever plates. The occurrence of consistent trends in both the reduction of frequency values and the distortions of mode shapes confirms the sensitivity of modal parameters with respect to the location and size of cracks."

"The above findings are of paramount importance in the establishment of non-invasive diagnostic methodologies for Structural Health Monitoring, since the variations in the modal values are ideally the primary indicators of damage."

Furthermore, dual validation method being empirical and numerical provides a sound framework to generalize this problem solution to a more complex scenario involving geometries and materials (like composites), and multiple cracks.

Also, an empirical formula was developed through the calibration process, utilizing equations (4, 6, and 7), based on a comparison and calibration with the results obtained via the FEM, as presented in table (1). The corresponding values of the calibration constants α and γ for each mode are summarized in table (3), along with the coefficient of determination R^2, which reflects the goodness of fit. These empirical formulation enable the estimation of the reduction in the first six natural frequencies of the idealized homogeneous plate investigated in the current study. Moreover, they can be generalized and applied to any other plate composed of a homogeneous material, thus broadening their applicability beyond the specific configuration analyzed here.

Table 3. Calibration Constants and Coefficient of Determination for Each Mode

Mode	α	γ	R^2
1	0.75277	1.92784	0.99999
2	1.01079	1.77100	0.99918
3	0.13941	1.33462	0.99537
4	0.97138	1.96395	0.99934
5	0.28587	1.28937	0.99426
6	1.63409	2.01637	0.99846

5. Conclusion

This study presented a comprehensive investigation into the effects of an edge crack at the fixed end of a cantilever steel plate on its vibrational behavior, focusing on natural frequencies and mode shapes. A dual methodology combining numerical simulation using the finite element method (FEM) via ANSYS APDL and calibrated analytical empirical formula was employed to quantify the influence of crack length on the dynamic response,

The results revealed a clear and consistent reduction in the first six natural frequencies with increasing crack length-to-width ratio (a/W). the fundamental frequency was found to be the most sensitive to crack growth, exhibiting a reduction of over 10% for the largest crack configuration (a/W = 0.5). Higher-order modes also experienced varying degrees of degradation, depending on the proximity of modal deformation near the clamped boundary, as well as offering further evidence of reduced stiffness in the locale.

A good level of conformity of the numerical results of analysis with the experimental data has been ensured, thus validating the finite element analysis methodology as well as the developed empirical formula. These results have strengthened the scope of vibration analysis in terms of its viability as a non-intrusive methodology for damage identification of a structure.

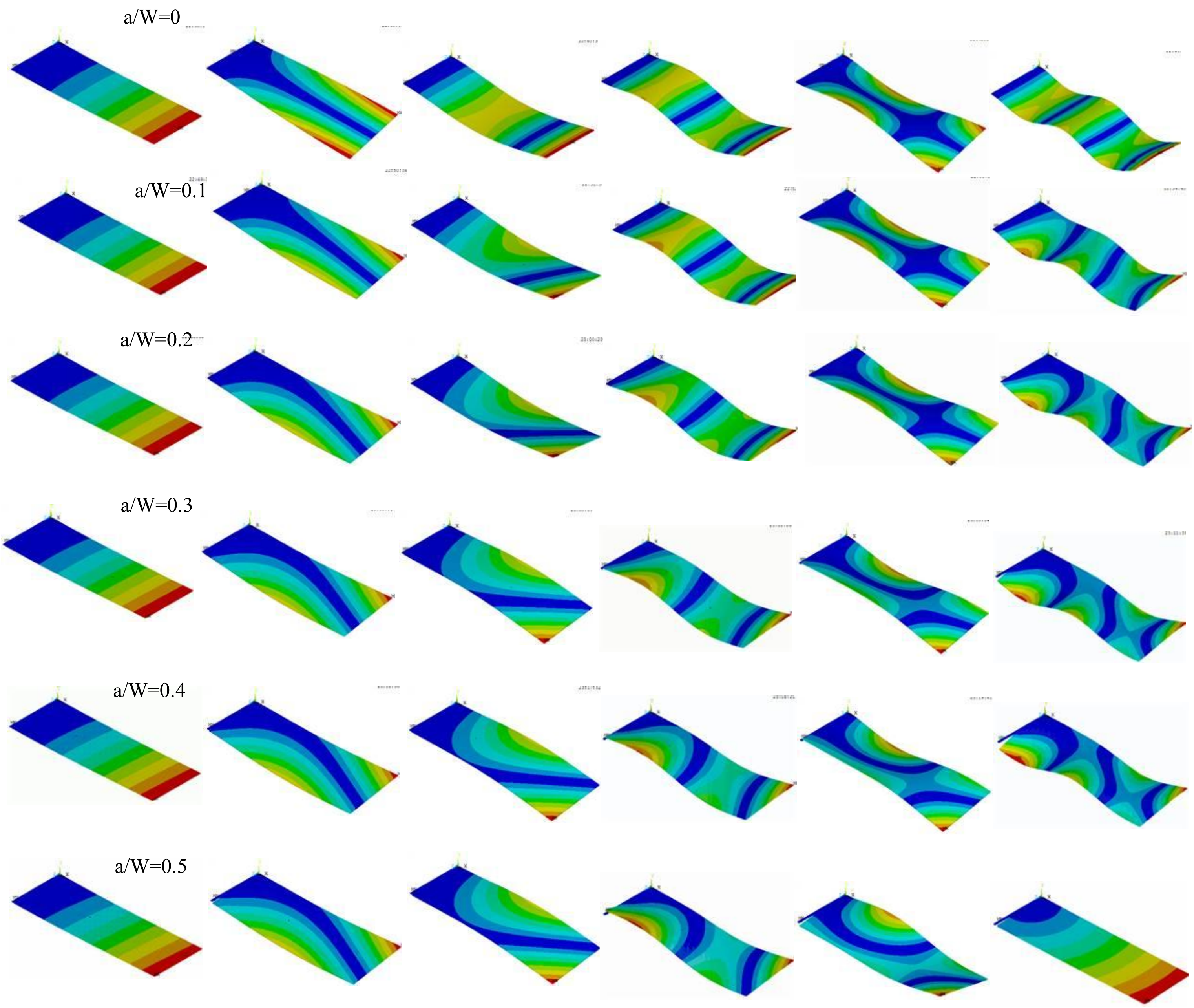

Fig. 5. Evolution of Mode Shapes as a Function of Increasing a/W.

This investigation has proved the high sensitivity of natural modes as well as natural frequencies to the cracked length in the clamped end, has provided quantitative as well as graphical proofs for vibration degradation caused by severe cracking, has confirmed the correctness of FEM calculations using ANSYS, and has also highlighted the capabilities for early diagnostic applications using modal analysis as a powerful tool for diagnostic purposes for components.

The obtained empirical formula, as well as their data, form a significant reference frame for future research concerning Structural Health Monitoring (SHM) applications and can be easily used for early damage identification for structural components. This work improves the prediction abilities concerning the vibration response for damaged systems and can represent a starting-point tool for research purposes as well as for engineering applications.

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